Rules and Principles used in

*The Many Worlds of Logic*

1. Truth-Functional Logic

(Chapters 2-11 of *The Many Worlds of Logic*)

**Truth-Tables**

<table>
<thead>
<tr>
<th>Conjunction</th>
<th>Disjunction</th>
<th>Conditional</th>
</tr>
</thead>
<tbody>
<tr>
<td>P Q</td>
<td>P &amp; Q</td>
<td>PQ</td>
</tr>
<tr>
<td>T T</td>
<td>T</td>
<td>TT</td>
</tr>
<tr>
<td>T F</td>
<td>F</td>
<td>TF</td>
</tr>
<tr>
<td>F T</td>
<td>F</td>
<td>FT</td>
</tr>
<tr>
<td>F F</td>
<td>F</td>
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</tr>
</tbody>
</table>

**Biconditional**

<table>
<thead>
<tr>
<th>PQ</th>
<th>P ≡ Q</th>
<th>P</th>
<th>~P</th>
</tr>
</thead>
<tbody>
<tr>
<td>TT</td>
<td>T</td>
<td>T</td>
<td>F</td>
</tr>
<tr>
<td>TF</td>
<td>F</td>
<td>F</td>
<td>T</td>
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<tr>
<td>FT</td>
<td>F</td>
<td></td>
<td></td>
</tr>
<tr>
<td>FF</td>
<td>T</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Logical Status**

- A formula is a truth-functional *tautology* if and only if the final column of its truth-table is all T’s.
- A formula is a truth-functional *contradiction* if and only if the final column of its truth-table is all F’s.
- A formula is truth-functionally *contingent* if and only if the final column of its truth-table contains at least one T and at least one F.
- An argument is truth-functionally *valid* if and only if its truth-table contains no row with all true premises and a false conclusion.
- An argument is truth-functionally *invalid* if and only if its truth-table contains at least one row with all true premises and a false conclusion.
- Two formulas are truth-functionally *equivalent* if and only if the final columns on their respective truth-tables match.
## Truth-functional Inference Rules

<table>
<thead>
<tr>
<th>Rule</th>
<th>Premises</th>
<th>Conclusion</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Disjunctive Syllogism (DS)</strong></td>
<td>From: ( P \lor Q ) and ( \neg P )</td>
<td>You may infer: ( Q )</td>
</tr>
<tr>
<td><strong>Modus Ponens (MP)</strong></td>
<td>From: ( P \supset Q ) and ( P )</td>
<td>You may infer: ( Q )</td>
</tr>
<tr>
<td><strong>Modus Tollens (MT)</strong></td>
<td>From: ( P \supset Q ) and ( \neg Q )</td>
<td>You may infer: ( \neg P )</td>
</tr>
<tr>
<td><strong>Hypothetical Syllogism (HS)</strong></td>
<td>From: ( P \supset Q ) and ( Q \supset R )</td>
<td>You may infer: ( P \supset R )</td>
</tr>
<tr>
<td><strong>Simplification (Simp)</strong></td>
<td>From: ( P \land Q )</td>
<td>You may infer: ( P ) and: ( Q ) and: ( Q )</td>
</tr>
<tr>
<td><strong>Conjunction (Conj)</strong></td>
<td>From: ( P ) and: ( Q ) and: ( Q \supset R )</td>
<td>You may infer: ( P \land Q )</td>
</tr>
<tr>
<td><strong>Addition (Add)</strong></td>
<td>From: ( P ) and: ( R \supset S ) and: ( P \supset R )</td>
<td>You may infer: ( Q \lor S )</td>
</tr>
<tr>
<td><strong>Constructive Dilemma (CD)</strong></td>
<td>From: ( P \supset Q ) and: ( R \supset S ) and: ( P \lor R )</td>
<td>You may infer: ( Q \lor S )</td>
</tr>
<tr>
<td><strong>Rule of Indirect Proof (IP)</strong></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Anywhere in a proof, you may indent, assume \( \sim P \), derive a contradiction, end the indentation, and assert \( P \).

**Rule of Conditional Proof (CP)**

Anywhere in a proof, you may indent, assume \( P \), derive \( Q \), end the indentation, and assert \( P \implies Q \).

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**Truth-functional Replacement Rules**

**Commutation (Comm)**

A formula \( P \lor Q \) may replace or be replaced by the corresponding formula \( Q \lor P \).

A formula \( P \land Q \) may replace or be replaced by the corresponding formula \( Q \land P \).

**Association (Assoc)**

A formula \((P \lor Q) \lor R\) may replace or be replaced with the corresponding formula \( P \lor (Q \lor R) \).

A formula \((P \land Q) \land R\) may replace or be replaced with the corresponding formula \( P \land (Q \land R) \).

**Double Negation (DNeg)**

A formula \( \sim \sim P \) may replace or be replaced with the corresponding formula \( P \).

**DeMorgan (DM)**

Algorithm:

1. Change the ampersand to a wedge or the wedge to an ampersand.
2. Negate each side of the ampersand or wedge.
3. Negate the formula as a whole.

Or:
A formula \( \neg (P \lor Q) \) may replace or be replaced by the corresponding formula \( \neg P \land \neg Q \).

A formula \( \neg (P \land Q) \) may replace or be replaced by the corresponding formula \( \neg P \lor \neg Q \).

A formula \( (P \lor Q) \) may replace or be replaced by the corresponding formula \( \neg (\neg P \land \neg Q) \).

A formula \( (P \land Q) \) may replace or be replaced by the corresponding formula \( \neg (\neg P \lor \neg Q) \).

**Distribution (Dist)**

A formula \( P \lor (Q \land R) \) may replace or be replaced with the corresponding formula \( (P \lor Q) \land (P \lor R) \).

A formula \( P \land (Q \lor R) \) may replace or be replaced with the corresponding formula \( (P \land Q) \lor (P \land R) \).

**Transposition (Trans)**

A formula \( P \Rightarrow Q \) may replace or be replaced with the corresponding formula \( \neg Q \Rightarrow \neg P \).

**Implication (Imp)**

A formula \( P \Rightarrow Q \) may replace or be replaced with the corresponding formula \( \neg P \lor Q \).

**Exportation (Exp)**

A formula \( (P \land Q) \Rightarrow R \) may replace or be replaced with the corresponding formula \( P \Rightarrow (Q \Rightarrow R) \).

**Tautology (Taut)**

A formula \( P \) may replace or be replaced with the corresponding formula \( P \lor P \).
**Equivalence (Equiv)**

A formula $P \equiv Q$ may replace or be replaced with the corresponding formula $(P \Rightarrow Q)$ & $(Q \Rightarrow P)$.

A formula $P \equiv Q$ may replace or be replaced with the corresponding formula $(P \& Q)$ $\lor (\sim P \& \sim Q)$.

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**2. Categorical (Aristotelian) Logic**

*(Chapters 14-15 of* The Many Worlds of Logic*)

<table>
<thead>
<tr>
<th>Sentence Type:</th>
<th>Logical Form:</th>
</tr>
</thead>
<tbody>
<tr>
<td>A sentence (Universal Affirmative):</td>
<td>All S are P</td>
</tr>
<tr>
<td>E sentence: (Universal Negative):</td>
<td>No S are P</td>
</tr>
<tr>
<td>I sentence (Particular Affirmative)</td>
<td>Some S are P</td>
</tr>
<tr>
<td>O sentence: (Particular Negative):</td>
<td>Some S are not P</td>
</tr>
</tbody>
</table>

S: subject term  P: predicate term

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**THE TRADITIONAL SQUARE OF OPPOSITION**

Converse, Obverse, Contrapositive
To Produce the *Converse* of a Categorical Statement:

Switch the subject and predicate terms.

To Produce the *Obverse* of a Categorical Statement:

1. Change the quality (without changing the quantity) from affirmative to negative or negative to affirmative.
2. Replace the predicate term with its term complement.

To Produce the *Contrapositive* of a Categorical Statement:

1. Switch the subject and predicate.
2. Replace each term with its term complement.

Equivalence

Equivalence relations between opposing categorical statements:

<table>
<thead>
<tr>
<th>Statement</th>
<th>Converse</th>
</tr>
</thead>
<tbody>
<tr>
<td>A: All S are P</td>
<td>All P are S</td>
</tr>
<tr>
<td>E: No S are P</td>
<td>No P are S</td>
</tr>
<tr>
<td>I: Some S are P</td>
<td>Some P are S</td>
</tr>
<tr>
<td>O: Some S are not P</td>
<td>Some P are not S</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Statement</th>
<th>Obverse</th>
</tr>
</thead>
<tbody>
<tr>
<td>A: All S are P</td>
<td>No S are non-P</td>
</tr>
<tr>
<td>E: No S are P</td>
<td>All S are non-P</td>
</tr>
<tr>
<td>I: Some S are P</td>
<td>Some S are not non-P</td>
</tr>
<tr>
<td>O: Some S are not P</td>
<td>Some S are non-P</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Statement</th>
<th>Contrapositive</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>
3. Quantificational Logic

(Chapters 16-22 of *The Many Worlds of Logic*)

Quantificational Inference Rules

**Universal Instantiation (UI)**

From a universal quantification, one may infer any instantiation, provided that the instantiation was produced by uniformly replacing each occurrence of the variable that was bound by the quantifier with a constant or John Doe name.

**Existential Generalization (EG)**

From a sentence containing a constant or a John Doe name, you may infer any corresponding existential generalization, provided that: (a) the variable used in the generalization does not already occur in the sentence generalized upon; (b) the generalization results by replacing at least one occurrence of the constant or John Doe name with the variable, and no other changes are made.

**Existential Instantiation (EI)**
From an existential quantification, you may infer an instantiation, provided that (a) each occurrence of the variable bound by the quantifier in the existential quantification is uniformly replaced with a John Doe name and no other changes are made; (b) the John Doe name does not appear in any earlier line of the deduction.

**Universal Generalization (UG)**

From a sentence containing a John Doe name, one may infer the corresponding universal generalization, provided that (a) the John Doe name that is replaced by a variable does not occur in any preceding line derived by EI, (b) the generalization results by replacing each occurrence of the John Doe name with the variable (and no other changes are made), (c) the variable you use in the generalization does not already appear in the sentence you are generalizing from, (d) the John Doe name does not appear in any assumed premise that has not already been discharged.

**Quantificational Replacement Rules**

**Quantifier Exchange (QE)**

If $P$ contains either a universal or an existential quantifier, $P$ may be replaced by or may replace a sentence that is exactly like $P$ except that one quantifier has been switched for the other in accord with the (a) - (c) below:

(a) switch one quantifier for the other.

(b) negate each side of the quantifier

(c) cancel out any double negatives that result.
Identity

Identity A (Id A)

At any step in a proof, you may assert (x) (x = x).

Identity B (Id B)

If c and d are two constants or John Doe names and a line of a proof asserts that the individual designated by c is identical with the individual designated by d, then you may carry down and rewrite any available line of the proof replacing any or all occurrences of c with d or any or all occurrences of d with c. A line of a proof is available unless it is within the scope of a discharged assumption.

4. Modal Logic

(Chapters 23-24 of The Many Worlds of Logic)

Modal Inference Rules

Box Removal (BR): From a sentence □ P, you may infer the corresponding sentence P.

Possibilization (Poss): From a sentence P you may infer the corresponding sentence ◇ P.

Modal Modus Ponens (MMP): From P → Q and the corresponding sentence P you may infer the corresponding sentence Q.

Modal Modus Tollens (MMT): From P → Q and the corresponding sentence ~Q, you may infer the corresponding sentence ~P.
Modal Hypothetical Syllogism (MHS): From $P \rightarrow Q$ and the corresponding sentence $Q \rightarrow R$ you may infer the corresponding sentence $P \rightarrow R$.

The Possibility to Necessity Rule (P to N)

From: $P \rightarrow Q$
and $\Diamond P$

You may infer: $\Box Q$  Provided that: the formula instantiating Q is itself a modally closed formula.

The Necessitation Rule (Nec)

At any point in a proof, you may indent and construct a "necessitation subproof" in which every line is either justified by the reiteration rule (below) or follows from previous lines of the subproof by a valid rule of inference. You may then end the indentation, draw a line around the indented lines, write down any line derived within the subproof, and then prefix a box to that line. (Write as justification "Nec" and the line numbers of the subproof.)

Reiteration

You may reiterate into a necessitation subproof any line provided that the entire line consists of just one modally closed formula and the formula does not lie within the scope of a discharged assumption or a terminated nec intro subproof. (Write as justification "Reit.")

Tautology Necessitation (Taut Nec)

If a statement $P$ is proven tautological, we may infer from this a statement $\Box P$. (Write as justification “Taut Nec” and the line the tautology appears on.)

Modal Replacement Rules
**Hook Conversion (HC):**

A sentence of the form $P \rightarrow Q$ may replace or be replaced with the corresponding sentence $\Box (P \supset Q)$.

**Double Hook Conversion (DHC):**

A sentence of the form $P \leftrightarrow Q$ may replace or be replaced with the corresponding sentence $\Box (P \equiv Q)$.

**Modal Equivalence (Mod Equiv):**

A sentence of the form $[(P \rightarrow Q) \& (Q \rightarrow P)]$ may replace or be replaced with the corresponding sentence $\Box (P \leftrightarrow Q)$.

**Diamond Exchange (DE):**

If a sentence $P$ contains either a box or a diamond, $P$ may be replaced by or may replace a sentence that is exactly like $P$ except that the box has been switched for the diamond or vice versa in accord with the (a) - (c) below:

- a. Add a tilde to each side of the box or diamond.
- b. Trade the box for a diamond or the diamond for a box.
- c. Cancel out any double negatives that result.

**Reduction (Red)**

Any sequence of iterated monadic modal operators in a formula may be reduced to the last member on the right, and the resulting reduced formula may replace the original formula anywhere within a proof. (Write as justification "Red" and the line of the reduced formula.)

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**5. Definition**

*(Chapter 12 of *The Many Worlds of Logic*)
**Definition by genus and difference** An important type of analytic definition in which a species or kind of entity is defined in two steps: a) we specify a general class or genus to which all the objects in the species belong; and (b) we narrow this down by indicating how this species differs from other species in the same genus.

**Extensional (or “denotative”) definition** A definition that assigns meaning to a word or phrase by giving examples of what the word or phrase denotes. There are three types:

**Enumerative definitions** assign meaning by naming or listing members of the extension.

**Ostensive (or “demonstrative”) definitions** involve pointing or gesturing at an item belonging to the extension.

**A definition by subclass** assigns meaning by naming or listing subclasses of the class of entities denoted by a term.

**Extensional (or “denotative”) meaning of a term** The class of objects to which the term may correctly be applied, that is, the members of the class that the term denotes.

**Intensional (or “connotative”) definition** A definition that assigns meaning by indicating the qualities or attributes a word or phrase connotes, that is, by listing the properties that an entity must have if the word or phrase is to apply to it. There are three common types:

A **synonymous definition** assigns meaning to a word by providing a synonym.

An **operational definition** assigns meaning to a word or phrase by specifying an operation or set of procedures that determines whether the word or phrase is applied to an entity or not.

An **analytical definition** attempts to explain the meaning of a word or phrase by specifying the characteristics possessed in common by those items to which the word or phrase applies.

**Intensional (or “connotative”) meaning** The qualities or attributes the term connotes, that is, the common attributes or characteristics that lead us to apply the term.

**Lexical meaning** The commonly understood meaning of a word or phrase.

**Lexical Definition** A definition that reports a word’s commonly understood meaning.

**Persuasive Definition** A definition that aims to influence attitudes.
**Precising Definition** A definition that provides a more precise meaning for a word that formerly had a vague but established meaning. The more precise meaning provides additional guidance as to how the word is to be applied in various borderline cases.

**Stipulative Definition** A definition that constitutes a new meaning for a word or phrase.

**Theoretical Definition** A definition that characterizes the nature of something. Such a definition provides a theoretical picture of an entity, that is, a way of understanding the entity.

### 6. A Summary of the Fallacies

*(Chapter 13 of *The Many Worlds of Logic*)

**Fallacies of No Evidence**

**Argument Against the Person** (argumentum ad hominem) This fallacy is committed when you attack a person’s character or personal circumstances in order to oppose or discredit their argument or viewpoint. Also:

**Tu Quoque Fallacy** (“you’re one, too”) A type of abusive ad hominem that attempts to discredit a person’s viewpoint or position by charging the person with hypocrisy or inconsistency. Essentially, the charge is, “We don’t need to take his argument seriously because he doesn’t practice what he preaches.”

**Guilt by association Fallacy** A type of abusive ad hominem in which one person attacks a second person’s associates in order to discredit the person and thereby his view or argument.

**Appeal to Force** (argumentum ad baculum, literally “argument from the stick”) A fallacy committed when an arguer appeals to force or to the threat of force to make someone accept a conclusion.

**Appeal to Pity** (argumentum ad misericordiam) A fallacy committed when the arguer attempts to evoke pity from the audience and tries to use that pity to make the audience accept a conclusion.

**Appeal to the People** (argumentum ad populum) A fallacy committed when an arguer attempts to arouse and use the emotions of a group to win acceptance for a conclusion.

**Snob Appeal Fallacy** This is committed when the arguer claims that if you will adopt a particular conclusion, this will place you in a special, elite group or will make you better than everyone else.
Fallacy of Irrelevant Conclusion (ignoratio elenchi, meaning “ignorance of the proof”) A fallacy in which someone puts forward premises in support of a stated conclusion, but the premises actually support a different conclusion.

Begging the Question Fallacy (petitio principii, meaning “postulation of the beginning”) This is committed when someone employs the conclusion (usually in some disguised form) as a premise in support of that same conclusion.

Appeal to Ignorance (argumentum ad ignorantium) In this fallacy, someone argues that a proposition is true simply on the grounds that it has not been proven false (or that a proposition must be false because it has not been proven true).

Red Herring Fallacy A fallacy committed when the arguer tries to divert attention from his opponent’s argument by changing the subject and drawing a conclusion about the new subject.

Genetic Fallacy A fallacy committed when someone attacks a view by disparaging the view’s origin or the manner in which the view was acquired.

Poisoning the Well The use of emotionally charged language to discredit an argument or position before arguing against it.

Fallacies of Little Evidence

Fallacy of Accident A fallacy committed when a general rule is applied to a specific case, but because of extenuating circumstances, the case is an exception to the general rule and the general rule should not be applied to the case.

Straw Man Fallacy A fallacy committed when an arguer (a) summarizes his opponent’s argument but the summary is an exaggerated, ridiculous, or oversimplified representation of the opponent’s argument that makes the opposing argument appear illogical or weak; (b) the arguer refutes the weakened, summarized argument; and (c) the arguer concludes that the opponent’s actual argument has been refuted.

Appeal to Questionable Authority Fallacy (argumentum ad verecundiam) When someone attempts to support a claim by appealing to an authority that is untrustworthy, or when the authority is unqualified, or prejudiced, or has a motive to lie.

Fallacy of Hasty Generalization A fallacy committed when someone draws a generalization about a group on the basis of observing an unrepresentative sample of the group.

False Cause Fallacy A fallacy involving faulty reasoning about causality. Also:

In a Post Hoc Ergo Propter Hoc fallacy (“after this, therefore, because of this”)
someone concludes that A is the cause of B simply on the grounds that A preceded B in time.

In a **Non Causa Pro Causa fallacy** (“not the cause for the cause”) someone claims that A is the cause of B, when in fact (1) A is not the cause of B, but (2) the mistake is not based merely on one thing coming after another thing. One version of this fallacy is the fallacy of accidental correlation: the arguer concludes that one thing is the cause of another thing from the *mere* fact that the two phenomena are correlated.

**Slippery Slope Fallacy** (or “domino argument”) In this fallacy, someone objects to a position P on the grounds that P will set off a chain reaction leading to trouble; but no reason is given for supposing the chain will actually occur. Metaphorically, if we adopt a certain position, we will start sliding down a slippery slope and we won’t be able to stop until we slide all the way to the bottom (where some bad result lies in wait).

**Fallacy of Weak Analogy** A fallacy committed when an analogical argument is presented but the analogy is too weak to support the conclusion.

**Fallacy of False Dilemma** A fallacy committed when someone assumes there are only two alternatives, eliminates one of these two, and concludes in favor of the second, when more than the two stated alternatives exist, but have not been considered.

**Fallacy of Suppressed Evidence** In this fallacy, evidence that would count heavily against the conclusion is left out of the argument or is covered up.

**Fallacy of Special Pleading** In this fallacy, the arguer applies a principle to someone else’s case but makes a special exception to the principle in his own case.

**Fallacies of Language**

**Fallacy of Equivocation** In this fallacy, a particular word or phrase is used with one meaning in one place, that word or phrase is used with another meaning in another place, and what has been established on the basis of the one meaning is regarded as established with respect to the other meaning. As a result, the conclusion depends on a word (or phrase) being used in two different senses in the argument. The premises are true on one interpretation of the word, but the conclusion follows only from a different interpretation.

**Fallacy of Amphiboly** A fallacy containing a statement that is ambiguous because of its grammatical construction. One interpretation makes the statement true, the other makes it false. If the ambiguous statement is interpreted one way, the premise is true but the conclusion is false; but if the ambiguous statement is interpreted the other way, the premise is false. The meaning must shift if the argument is going to go from a true premise to a true conclusion. If the meaning is not allowed to shift during the argument, either the argument has false a premise or it is invalid.
**Fallacy of Composition** A fallacy in which someone uncritically assumes that what is true of a part of a whole is also true of the whole.

**Fallacy of Division** A fallacy in which someone uncritically assumes that what is true of the whole must be true of the parts.

### 7. Induction

*(Chapters 25-26 of The Many Worlds of Logic)*

**Analogical argument**

An argument in which (a) an analogy is asserted between two things or kinds of things, X and Y; (b) it is then asserted that X has a particular feature and that Y is not known not to have the feature; (c) it is concluded that Y *probably* also has the feature. More formally:

1. X has features ABCD.
2. Y has features ABCD.
3. X also has feature E.
4. Y is not known not to have E.
5. Therefore, Y *probably* has feature E as well.

The more features in common, the stronger the argument. The higher the degree of causal or statistical relevance between the features cited in the analogy and the feature cited in the conclusion, the stronger the argument.

**Enumerative Induction**

An argument in which premises about observed individuals or cases are used as a basis for a generalization about unobserved individuals or cases.

In one variation, a **sample** of a group is observed and is found to have feature X. It is then concluded that the group *probably* also has feature X.

The more random the sample, the stronger the argument. The more heterogenous the sample, the stronger the argument. The larger the sample, the stronger the argument.

In another variation, a **series** is extended:

A is an F and has feature G
B is an F and has feature G
C is an F and has feature G
D is an F and has feature G
Therefore, the next F encountered will probably have feature G.

The more random the series, the stronger the argument. The longer the series, the stronger the argument.

Inference to the Best Explanation (also called “abduction”)

1. The argument cites one or more purported facts, which, it is claimed, are in need of explanation.
2. Possible explanations of the facts are considered.
3. It is argued that a particular explanation is the best or most reasonable explanation of the facts.
4. It is concluded that this explanation is probably the correct explanation.

Comparing explanations

- A good explanation is **internally consistent**—it contains no self-contradictory elements.
- A good explanation is **externally consistent**—it does not contradict already established facts and already proven theories.
- A good explanation explains the widest possible range of relevant data.
- The more completely the explanation explains the data, the better the explanation.
- If all else is equal, the potential explanation that explains more of the relevant data is preferable.
- If two potential explanations explain the same range of data and are otherwise equal, except that one explanation is **simpler** than the other, the simpler explanation is preferable. (One explanation is simpler than another if it makes reference to fewer entities or contains fewer explanatory principles or explanatory elements.)

Reason in Science
(Chapter 26)

Mill's Methods

Mill’s Methods is a set of principles formulated by the British philosopher John Stuart Mill (1806-73) in his *System of Logic* (1843) for use when seeking the probable cause of a circumstance or effect.
**Mill’s Method of Agreement** Draw up a list of possible causes. Try to find one causal factor common to all cases of the effect. If found, this is the *probable* cause or is part of the *probable* cause.

**Mill’s Method of Difference** Examine a case where an effect E occurs and a similar case where E does not occur. The probable cause is the one respect in which the case where the effect E occurs differs from the case where E is absent.

**Mill’s Method of Residues** If we know that (1) A, B, and C are causal conditions responsible for effects X, Y, and Z; and (2) A is found to be the cause of X; and (3) C is found to be the cause of Y, it follows that B, the residual factor, is probably the cause of Z.

**Mill’s Method of Concomitant Variation** If changes in one phenomenon accompany or correspond to (are concomitant with) changes in a second phenomenon, and if the magnitude of the change in the one varies along with the magnitude of the change in the second, the two phenomena are probably causally related—either one of the two probably causes the other, or some third factor is probably the cause of both.

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**Other Principles Used in Science**

**The Principle of Economy (or Occam’s Razor).** If two hypotheses explain the same range of data and are otherwise equal, except that one hypothesis is simpler than the other, the simpler hypothesis is preferable. One hypothesis is simpler than another if it makes reference to fewer entities or contains fewer explanatory principles or explanatory elements.

**To Conduct a Controlled experiment** (a) Two groups of individuals are compared; (b) the two groups must be extremely similar except for the following difference: one group, called the test group, has or is given the factor suspected to be the cause of the effect, and the other group, called the control group, is similar to the test group except that it lacks the factor under investigation, that is, the suspected cause; (c) the Method of Difference is employed to determine the probable cause.

**Confirmation of a Scientific Theory**

The process by which a scientific hypothesis is shown to be *probably* true:

1. An observational prediction (P) is derived from the conjunction of the hypothesis, data on initial conditions, and auxiliary assumptions.
2. Guided by the prediction, observations are made and the prediction is verified. P is found to be the case.
3. It is concluded that the hypothesis is probably true.

(An **observational prediction** is a prediction about the results of observations that can be made, where the observation can concern facts related to the past, present, or future.)

**Disconfirmation of a Scientific Theory**

The process by which a scientific hypothesis is shown to be false:

1. An observational prediction (P) is derived from the conjunction of the hypothesis, data on initial conditions, and auxiliary assumptions.
2. Guided by the prediction, observations are made and the prediction is found to be false. That is, P is found not to be the case.
3. It follows that the hypothesis, initial data, and assumptions cannot all be true.
4. The initial data and assumptions are confirmed as true.
5. It is concluded that the hypothesis must be false.